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LETTER TO THE EDITOR

Absence of many states in realistic spin glasses

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Abstract. Based on a simple scaling ansatz, we argue that no pure thermodynamic states other than the paramagnetic state and a pair of states in zero magnetic field which are related by a global spin flip can exist in short-range Ising spin glass models in *any* dimension. An analogous result should hold for XY and Heisenberg spin glasses, as well as for square-integrable long-range intractions.

One of the primary themes running through the theoretical literature on spin glasses is the idea that the ordered phase will generally be characterised by the coexistence of infinitely many pure (or extremal) states. This hypothesis arose from Parisi's solution of the infinite-range Sherrington–Kirpatrick (SK) model [1].

We have previously argued [2] that in *realistic* (i.e. finite-range) spin glasses, the nature of the ordered phase should be quite different from the SK model. For Ising systems in zero magnetic field and temperatures below the phase transition T_c, which should exist [2–5] for sufficiently large dimension, d, there are two pure states differing from one another by a global spin flip. In this letter we present several suggestive arguments, based on a scaling ansatz introduced in [2] that, in a strong sense, there do not exist any other pure states in realistic spin glasses. For most of this letter we concentrate on Ising systems and at the end discuss extensions of the conclusions to XY and Heisenberg systems. Note: we will henceforth drop 'pure' and use 'state' to mean a pure equilibrium state; we do not consider 'metastable states'. By definition, all other equilibrium states are linear combinations of the pure states [6].

Much of the discussion in the literature on the existence of many states in spin glasses has been focused on the 'overlap' function $P(q)$ [1]. In a companion letter [7] we demonstrated that $P(q)$ is not the appropriate quantity to address this issue for finite-range systems and, further, that in this regard both the SK model and spin glasses on Bethe lattices are likely to be very different from models on finite-dimensional lattices.

Instead of studying $P(q)$ we must ask whether, in a *given* realisation of an infinite Ising system, one can choose different sets of boundary conditions on an infinite sequence of boxes $\{B_m\}$, of linear sizes Λ_m (centred on the origin), in such a way as to yield sets of indecomposable correlation functions (states) near the origin which differ by other than a global spin flip. Two states differing by a global spin flip occur if the system orders in zero magnetic field; we are interested in the question of other possible states. In particular, can we find more than one thermodynamic limit for the nearest-neighbour spin correlation, $\langle S_0 S_1 \rangle$, between the spin at the origin and one of its nearest neighbours? For simplicity, we avoid exact degeneracies by restricting our attention to continuous distributions of exchange J_{ij}.

Physically, it is important to distinguish between (i) regionally congruent states which are related by a symmetry operation (i.e. spin flip) over all regions of the system except near defect structures (e.g. domain walls) of dimension less than d and (ii) incongruent states which differ by other than a spin flip over a finite fraction of the system. Examples of the distinction, discussed in detail in [7], are, respectively, (i) the up state and a state which consists of a single smooth domain wall separating up and down regions in the 3D Ising ferromagnet below its roughening temperature and (ii) coexisting paramagnetic and ferromagnetic states at a first-order phase transition, or, less trivially, the up and down states of a random-field ferromagnet [8].

Implicit in the literature on spin glasses appears to be the assumption that the finite-dimensional version of the SK model's many states, if they exist, will be incongruent in the above sense. Here we argue that *neither* incongruent states *nor* regionally congruent states exist other than a pair of states which differ by a *global* spin flip. (Two incongruent and dissimilar states could exist at isolated first-order transitions from, e.g., spin glass to paramagnet; we will henceforth exclude such special points from the discussion.)

The organisation of this letter is as follows: we first consider the behaviour at zero temperature of ground states in an infinite system. A ground state has the property that its energy cannot be lowered by flipping the spins in any *finite* region. We give the fundamental scaling ansatz for the 'stiffness' of the system as a function of length scale [2]. Then, based on this ansatz, we argue that no incongruent states exist. We next give two arguments based on the roughness of domain walls and on scale invariance that there do not even exist regionally congruent states except for the two *globally* congruent states which are the *unique* pair of states related by a global spin flip. The results are then extended to positive temperature. We discuss possible failures of the ansatz and their consequences, and, finally, experimental implications of our conclusions.

Our fundamental ansatz, motivated by the work of Anderson and Pond and others [4, 5], is that the stiffness of a spin glass ground state on length scale L scales as L^θ with $\theta > 0$ necessary in order to stabilise the spin glass phase at positive temperature. We restrict ourselves to this case. More precisely, for a d-dimensional system, consider a box of length L on each side with periodic boundary conditions in $d-1$ of the directions and a particular boundary condition a on the top and bottom of the box. The surface free energy (or at $T=0$ energy) between boundary conditions a and b is defined by

$$\sum_{ab} = \tfrac{1}{2}[F^a_a + F^b_b - F^a_b - F^b_a]$$

where F^a_b denotes the free energy with boundary condition a on the top of the box and b on the bottom. Of particular interest is the stiffness against spin flips, i.e. between a and $\bar{a}$ which is obtained from a by a global spin flip. Then we hypothesise, following [2], that $\Sigma_{a\bar{a}}$, which can be of either sign, typically has magnitude $\Sigma_{a\bar{a}} \sim L^\theta$ for $L \to \infty$. The exponent θ, which we have previously argued satisfies $\theta \leq (d-1)/2$, also controls the lowest energy droplet excitations of volume L^d in an infinite system [2]. It is also useful to define a maximum stiffness, $\tilde{\Sigma} = \max_{a,b} \Sigma_{ab}$, scaling as $L^{\tilde{\theta}}$ with $\tilde{\theta} \geq \theta$. The natural and simplest ansatz is that $\tilde{\theta} = \theta$ so that *any* spin glass stiffness on a scale L typically (in the distribution of realisations) scales as L^θ. We return at the end of the letter to possible consequences if $\tilde{\theta} \neq \theta$.

We first consider, in a given realisation, two putative incongruent ground states α, β which differ over a finite fraction of the system. On large length scales L, these differ by a positive fraction of the nearest-neighbour spin pairs in a positive fraction of boxes of size L^d. If the pairs were independent, the typical energy difference between α and β in such a box would be at least $L^{d/2}$. (In contrast, the fluctuations in a subset of L^x bonds with $x<d$ could be much less.) Anticorrelations due to the constraints that both α and β are ground states are unlikely to decrease the difference significantly; this is known to be the case for the up and down states in a random-field ferromagnet [8]. However, with a cost in energy of order $L^{\tilde{\theta}}$, a droplet of the locally lower energy state, say α, can be inserted into state β and connected to it on a scale $2L$. But since $\tilde{\theta}<\frac{1}{2}d$ this would lower the energy of state β. Thus incongruent ground states do not appear to be consistent with our ansatz. Note that it seems quite likely that the lowest energy configuration which differs from a ground state on a non-zero fraction of the bonds does, in fact, differ in energy *density*. Arguments for this would be extremely useful and would directly rule out incongruent states.

We now consider constructing states which differ from a given ground state, which we denote 1, and its global spin reversal $\bar{1}$ by domain walls.

Any configuration can be represented, up to a global spin flip, by a set, W, of $(d-1)$-dimensional domain walls between regions that are locally 1 and $\bar{1}$. Ground states consist of wall configurations, W^α, whose energy cannot be lowered by moving or removing finite sections of domain wall.

The simplest kind of ground state that is neither 1 or $\bar{1}$ consists of a single infinite wall, possibly with a complicated topology, passing across the system. This can be obtained by boundary conditions which fix the intersections, ∂W_m^α, of the wall, W^α, and the surfaces of boxes $\{B_m\}$. We must ask whether in the thermodynamic limit there is a positive probability that such boundary conditions can be chosen so as to make the minimum energy (minimal) wall, W^α, pass between sites zero and one. We argue below that this is *not* the case, and therefore that there are only two ground states, which differ simply by a global spin flip.

Because $\theta<d-1$, the energy of a section of a minimal domain wall on length scale L is not simply the sum of positive energies of smaller sections of scale less than L. In fact, the local energy of a small section of a large wall is almost as likely to be negative as positive. The wall deviates from a simple flat geometry in order to pass through regions where the local wall energy is negative. Because of this, we expect the minimal wall to have overhangs, handles, etc, on all length scales and hence to be fractal with surface area scaling as L^{d_s}, where $d_s>(d-1)$ is the fractal dimension of the domain wall. By the above argument for the absence of incongruent states, it follows that minimal walls cannot be space-filling, and hence $d_s<d$. This implies that the probability of a wall with a given ∂W_m^α passing through the origin vanishes as the box size $\Lambda_m\to\infty$. However, we must ask whether we can force W to pass through the origin by adjusting the boundary condition ∂W_m.

Because the wall is rough on the scale of the box, the position of W in the interior of a box *cannot* be very precisely controlled by adjusting the boundary condition, ∂W_m. If the boundary condition is adjusted by continuously moving ∂W_m, then typically the parts of W well inside the interior of the box do not move at all until a point is reached where a new position, some fraction of the box size Λ_m away, becomes of equal energy. Beyond this point the minimal wall, W, jumps discontinuously to the new position. Thus there are only a few positions that the sections of the wall in the central portion of the box can be forced into. If this were not the case, then the minimal wall would evolve smoothly as the boundary conditions are varied and there would exist large sections of minimal walls, W and W', which are very close to one another and which have nearly identical local energy. This is *a priori* improbable since it would imply an anomalous density of low-energy excitations in the putative ground state formed by the single wall W.

We thus conclude that, because the walls are rough on the same scale as their size, the probability that *any* minimal wall in a box of size Λ_m passes between sites 0 and 1 goes to zero as $\Lambda_m \to \infty$.

This argument depended on the form of the ground state and its excitations. We now present another argument which explicitly uses the scale invariance suggested by the existence of a non-trivial exponent θ.

We consider in an infinite system a sequence of boxes of size $\Lambda_m = A^m$ ($A \geq 2$) and investigate the probability that the mth box is isolated from the nth ($n > m$) which occurs if there does *not* exist *any* boundary condition on box B_n which alters the ground-state configuration inside box B_m by other than an overall spin flip.

We now give a suggestive argument that the probability p_m that the $(m+1)$th box is isolated from the $(m+k)$th (with A and k to be chosen) tends to a positive constant as $m \to \infty$. Since for $n \gg m$, p_n is nearly independent of p_m, it follows from this that, with probability one, any given box is isolated from infinity and hence, in any finite region of an infinite system, the ground state is unique up to an overall spin flip.

It is useful to introduce schematic coarse-grained spins $\{\mu_i^m\}$ for blocks of volume Λ_m^d. These should be definable since we have already argued that there are no incongruent states. The original scaling ansatz suggests the existence of a fixed point Hamiltonian for interactions between the block spins on level m with an overall energy scale Λ_m^θ and thus, by analogy with the critical point for percolation [9], we expect probabilities of certain types of events to be scale invariant. For example, if we coarse grain to the mth level and put boundary conditions on the μ^m, the probability q_m that the $(m+1)$th box is isolated from the $(m+k)$th is independent of m for large m. For some positive fraction of the cases in which the μ^m in B_{m+1} are isolated, the original spins in B_{m+1} are also; thus B_{m+1} is isolated from the block spin boundary conditions on ∂B_{m+k} with a probability of some fixed fraction of q_m for large m.

The effects of tuning the boundary conditions on scales finer than Λ_m can be taken into account iteratively (see figure 1). First note that a boundary condition on the μ^j has a comparable effect far away from the boundary to a weaker boundary condition on the μ^{j+1} at some distance $C\Lambda_j$ further in from the original boundary. Then by iterating, we see that a boundary condition on scale 1 should be roughly equivalent

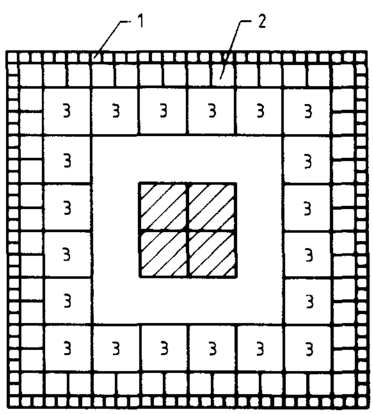

Figure 1. Illustration of a scale invariance argument for the probability of the shaded box being isolated from the boundary, showing coarse-grain spins at levels 1, 2 and 3 (see text).

to a boundary condition at scale Λ_m a distance $\Sigma_{j=0}^{m-1} CA^j < C\Lambda_m$ further in. This is illustrated in figure 1. But now, provided A^k is chosen to be greater than $4C$, the probability of isolating B_{m+1} from this new block spin boundary condition is again independent of m and hence $p_m \to$ a constant, as desired. We note that there is some chance of making an argument of this form more precise by choosing A and k to be large.

We have so far argued that, at $T=0$, any finite region of size Λ is isolated from boundary conditions far away. To extend the arguments to positive temperature, we coarse grain in the mth box to a scale $l_m \sim \Lambda_m^x$ with $x<1$ to take into account the thermal fluctuations as described in [2] (this only works for $\theta>0$). We then have effectively a ground-state problem, albeit with a markedly different Hamiltonian than would have been obtained from coarse graining the original $T=0$ problem. We then argue as above, but with isolation defined at each stage with some tolerance which can be chosen to approach zero for large boxes due to the low desnity, T/L^θ, of thermally activated fluctuations on large scales [2].

Our conclusion is that either $T_c=0$, which occurs for $\theta<0$, in which case there is only one state at all positive temperatures, or, for $\theta>0$, there are two states related by a global spin flip in zero magnetic fields for $T<T_c$ and, as argued in [2], one state elsewhere. For $\theta=0$, something more complicated could occur, but it is unlikely to involve more than two states at a given temperature.

For $\theta>0$, the states change with temperature, i.e. as T is changed the sign of the long distance correlations change randomly [2]. Associated with this—which essentially amounts to a continuous sequence of infinitesimal first-order phase transitions for $T<T_c$—is a breakdown of the conventional relationships between energy fluctuations and thermodynamic derivatives.

The arguments we have presented are based on the simplest possible ansatz for the scaling of the spin glass stiffness. However, it is possible that the behaviour is more complicated. In particular, a weak point is certainly the assumption that $\tilde{\theta}=\theta$. We briefly consider the consequences of $\tilde{\theta}>\theta$.

First note that, as long as $\tilde{\theta}\leq\frac{1}{2}d$, the arguments [2] for the absence of incongruent stattes are valid, since if incongruent states existed $\tilde{\theta}$ would be an upper bound for the scaling exponent $\hat{\theta}$ of the interfacial free energy between them. If $\tilde{\theta}>\frac{1}{2}d$, on the other hand, our arguments fail. Furthermore, our earlier arguments [2] for the absence of a transition in a magnetic field would fail since many states for $H>0$ could presumably be constructed from the incongruent states at $H=0$. However $\tilde{\theta}>\frac{1}{2}d$ (or more precisely $\hat{\theta}>\frac{1}{2}d$) does not necessarily imply the existence of incongruent states. In particular, as mentioned earlier, it may be possible to argue that, in general, *any* configuration which differs (by other than a spin flip) from a ground state in a finite fraction of space will differ in energy *density*, thus directly ruling out incongruent states.

In this letter, we have given several suggestive arguments that, for Ising spin glasses with short-range interactions, there are only two pure states which correspond to the broken global symmetry. As in [2], we expect the results to hold also for square integrable long-range spin glass interactions and for systems with continuous symmetry for which, when the symmetry is broken, the only possible states are globally related to one another by (possibly improper) elements of the rotation group. For XY or Heisenberg systems in a uniform magnetic field, it is still possible that the rotational symmetry about the field direction is spontaneously broken and there can thus be a Gabay-Toulouse line [10]. A randomly oriented magnetic field destroys the transition, however, as for the Ising case. Thus real spin glass ordered phases are, perhaps, rather similar to Edwards and Anderson's original picture [1] but radically different from Parisi's solution [1] of the SK model.

Note that the experimental consequences of the existence, or lack thereof, of many states are rather minimal, primarily because equilibrium on long length scales requires prohibitively long times and real systems are therefore always in a metastable state. However, in principle, microprobes which measure the detailed correlations in a small region of a large system might be used to address this question.

Bovier and Frohlich [12] have recently discussed the possibility of 'domain wall' states in short-range spin glasses; they reach conclusions rather different from ours. The fundamental discrepancy is associated with their assertion that an exponent related to θ must be equal to $(d-1)$ for a phase transition to exist.

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